Technical Note 1970-9

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On Estimating Explosive Source Parameters at Teleseismic Distances

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Lincoln Laboratory

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Lexington, Massachusetts

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LINCOLN LABORATORY

ON ESTIMATING EXPLOSIVE SOURCE PARAMETERS AT TELESEISMIC DISTANCES

J. R. FILSON

Group 22

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ABSTRACT

A study has been made of the short period spectra of five presumed explosions recorded at five arrays. An attempt has been made to relate contrasts in spectra of different events recorded at the same site, to source size; and contrasts observed at different arrays for a given event, to the earth's attenuative properties. Haskell's model for the explosion spectrum was fitted to each event individually after corrections for instrument response and various exponential attenuations. At a single array, that attenuation which allowed the fitted parameters to vary as dictated by the model was chosen as the correct one. With the attenuation estimated to each array, the spectra observed at all the arrays for a single event are fitted to a source model simultaneously. In most cases the individual and simultaneous fitting schemes yield reasonable values for the source parameters. Haskell's model and the estimated attenuation parameter for a central Asia to LASA path apparently explains a trend in short period spectral ratio measurements as a function of magnitude.

Accepted for the Air Force
Franklin C. Hudson
Chief, Lincoln Laboratory Office

I. INTRODUCTION

This report discusses certain features of the short period spectrum from five presumed nuclear explosions recorded at teleseismic distances at five array sites. The purpose of the study is to determine what reliable information concerning the nature of the source is to be gained from such data.

Source studies using the short period spectrum are difficult because source information is obscured, in the most simple case, by modulation within the source and receiver crustal regions and by attenuation within the crust and mantle. In this study, the detail of the source and receiver crustal regions is considered unknown and moreover, the effect of attenuation in the earth's mantle is so ill established that no *a priori* correction for this effect can be made.

The method of interpretation attempted here is therefore a simple and straightforward one. An assumed source model is fitted to each spectrum which is corrected for various exponential attenuations. That attenuation which shows the parameters of the model varying as theoretically predicted for events of a different size recorded at a single site is chosen as the estimate of the attenuation to that site. Once the attenuation is estimated at each site, the spectra of a single event recorded at five sites are fitted simultaneously to a single source model. Obviously results from such a technique cannot be taken as simultaneous verification of a given source model and absolute determination of the attenuation properties of the earth, but it does allow quantification of gross spectral differences observed between different events recorded at the same site and between the same event recorded at different sites.

II. THE DATA

The arrays at which the data was recorded vary widely in geographical position and size. The arrays are: the Large Aperture Seismic Array (LASA) in eastern Montana, U.S.A.; the United Kingdom arrays at Yellowknife, Canada (YKA); Gauribidanur, India (GBA); and Warramunga, Australia (WRA); and a temporary array operated by the United States near Oslo, Norway (OONW). The geometry and coordinates of the LASA subarrays are shown in Figure 1, those of the UK and Norway arrays are shown in Figure 2. LASA, extending over 200 km, consists of 21 subarrays of 25 seismometers each. The UK arrays, about an order of magnitude smaller, are made up of some 20 seismometers evenly spaced along two orthogonal arms of about 20 km length. The Norway array consists of seven seismometers, 1 km apart in an x-shaped pattern, extending over about 5 km. The relative shape of the response curves of the seismometers at each array are shown in Figure 3.

At each array for each event all seismometer, or as at LASA, all center subarray seismometer traces were aligned by eye and summed. The spectra of these beams were computed using time windows of 3 seconds and 10 seconds, both beginning about .5 seconds before the initial P motion. The above operations were done at the Lincoln Laboratory Data Analysis Console described by Fleck[1]. These beam spectra comprise the data set upon which all further analysis is based. Two arguments are made for using beam spectra in source studies. Firstly, the signal to noise ratio of the array beam is usually higher than that of an individual seismometer. Secondly, minimization of the effects of minor crustal variations within the array by summing is sought. Of course, the effectiveness of this latter attempt will depend upon the array size and the scale of the crustal variations beneath each array.

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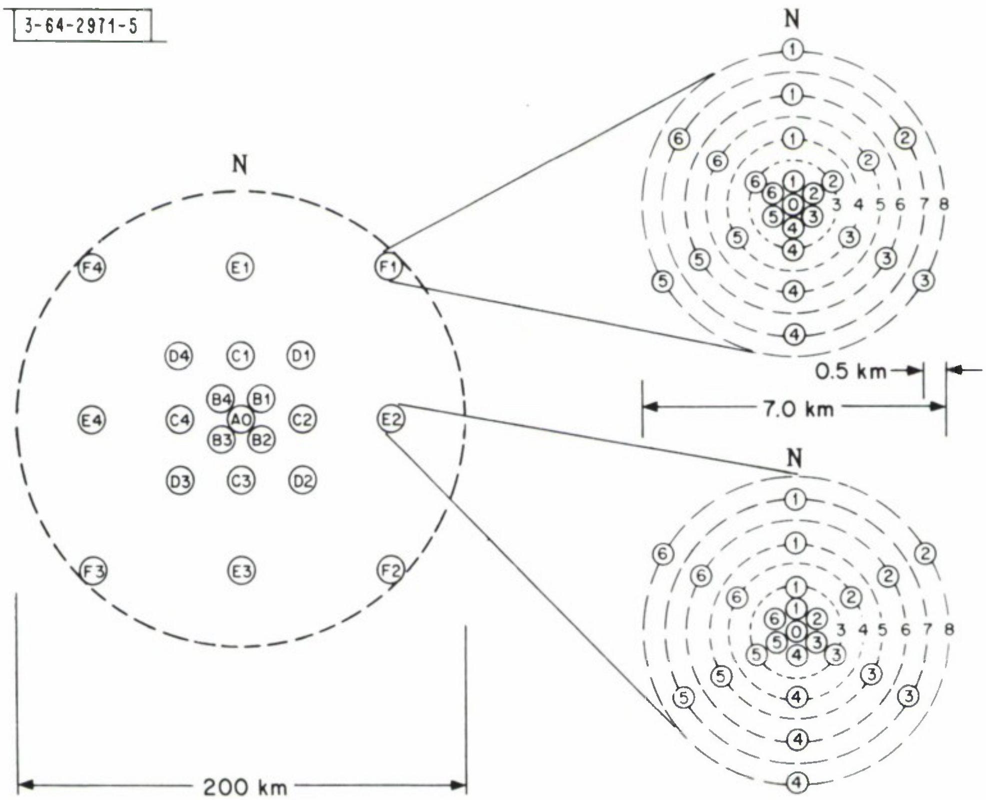

Fig. 1. LASA geometry. Center seismometer is at 46° 41' 19.0" N 106° 13' 20.0" W.

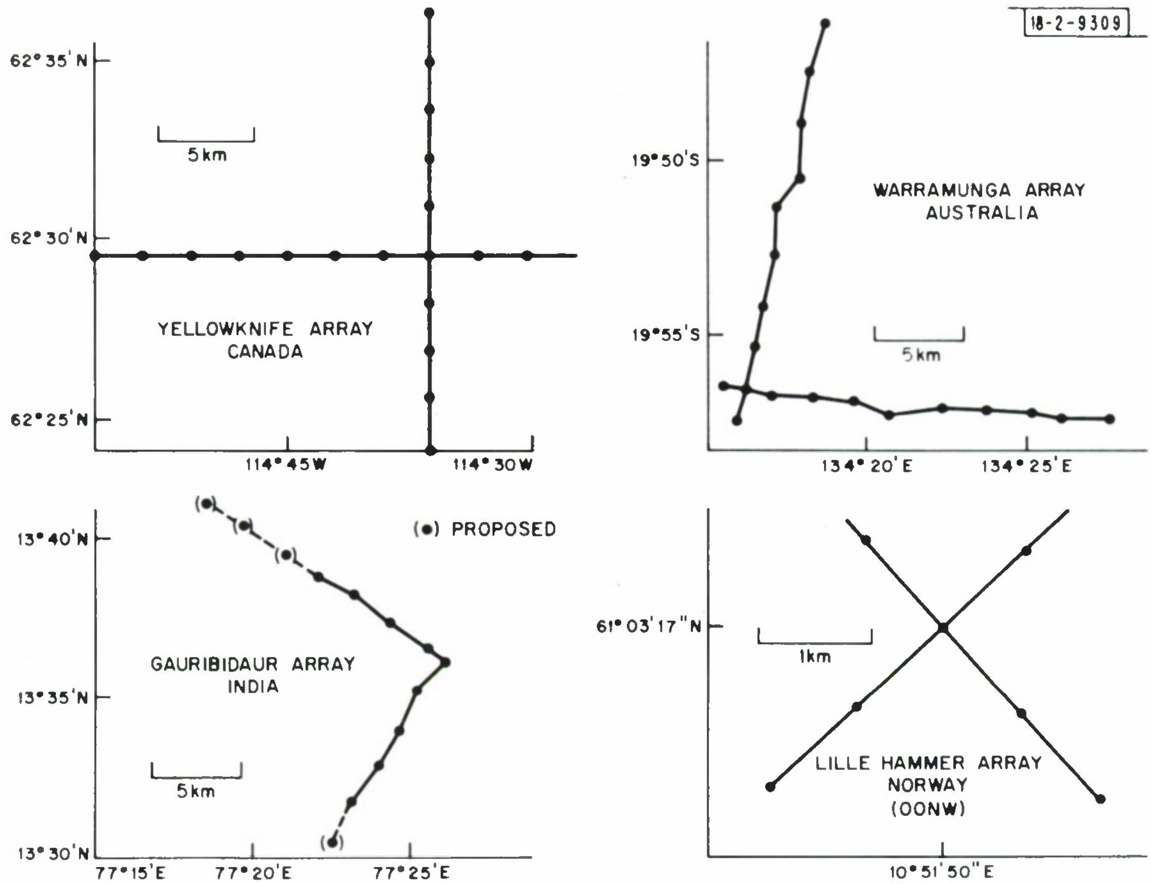

Fig. 2. Geometry of the United Kingdom arrays and OONW.

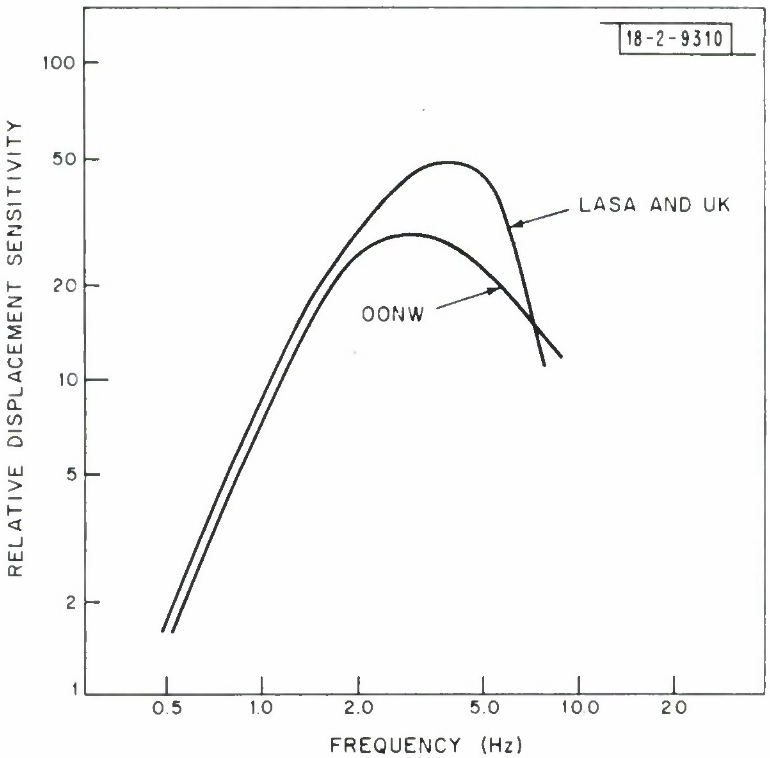

Fig. 3. Shapes of the response curves of the short period vertical instruments used in this study.

Figure 4 shows the global geometry of the experiment. The azimuthal variation is wide; OONW and GBA are about 38° from the source region (STS), YKA about 67°, LASA about 83°, and WRA about 87°.

The USCGS has reported 4 of the 5 presumed Soviet explosions used in this study. Figure 5 is a sketch map of the source region with the USCGS locations plotted and magnitudes listed. The magnitude of event 4 was estimated by the author from individual station bulletins. The locations using LASA are plotted and identified by the primed numbers in an attempt to establish a location error vector for the region and thus locate event 4 more accurately. Although the orientation of this error vector is established, its magnitude is not and the only conclusion to be drawn here is that event 4 should probably be located to the northwest of the LASA location.

Figures 6 and 7 show the beam traces at each array of events 1 and 5, the smallest and largest events studied. There are two points to be made here. The first is that the signal to noise ratio is quite good in both cases. The second is that variations in these time traces can be seen when beams of the two events recorded at a given array are compared. These variations are particularly strong at WRA and YKA. Since the events are tightly grouped geographically, these variations must either be attributed to the size and medium of the source or to the effects of a minor change in propagation path, or both.

The second spectra and 10 second spectra are shown in Figures 8 and 9 respectively. Plotted in these figures are the beam spectrum moduli, between 0.0

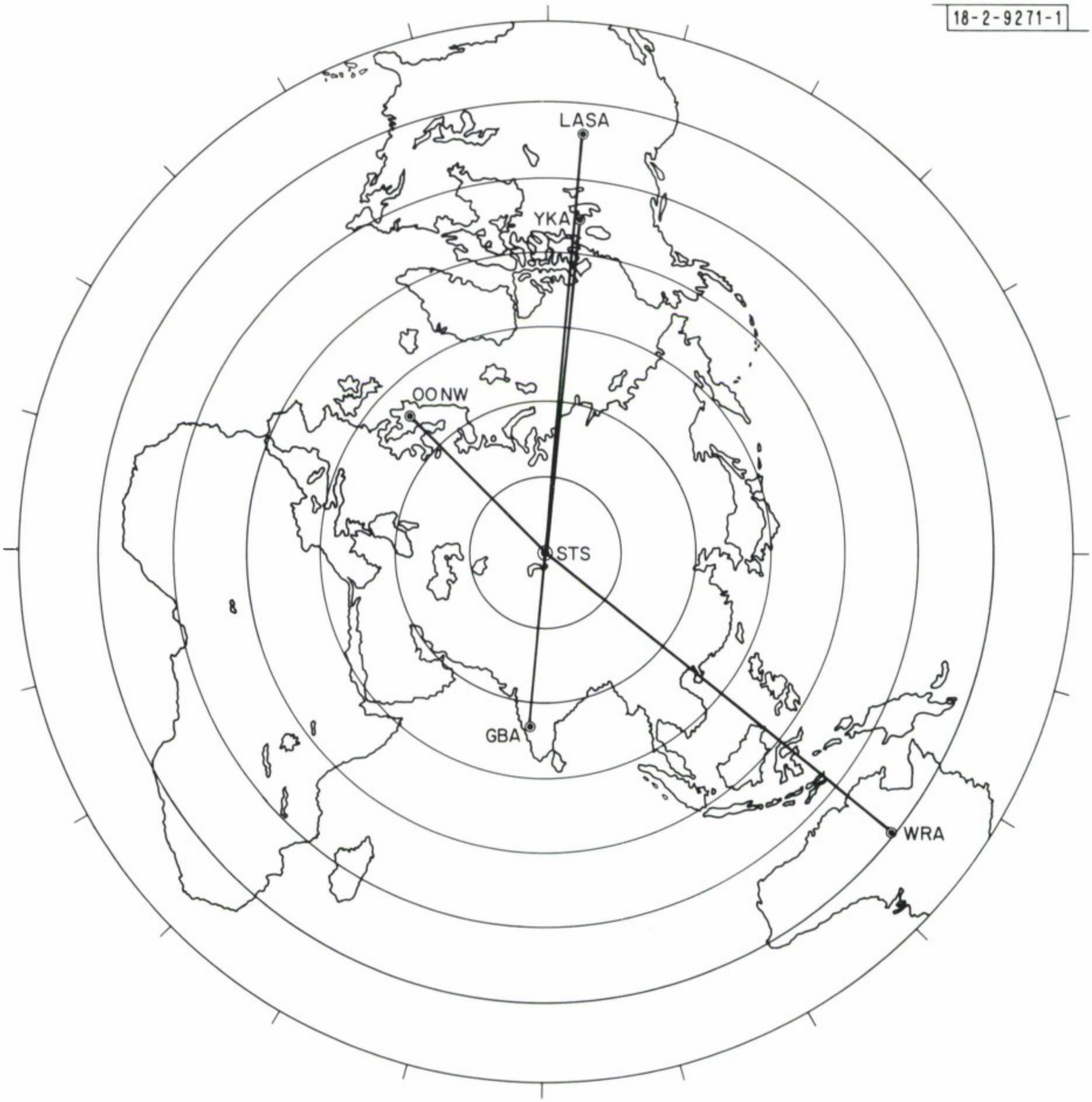

Fig. 4. World map showing the locations of the arrays with respect to the Soviet test site (STS).

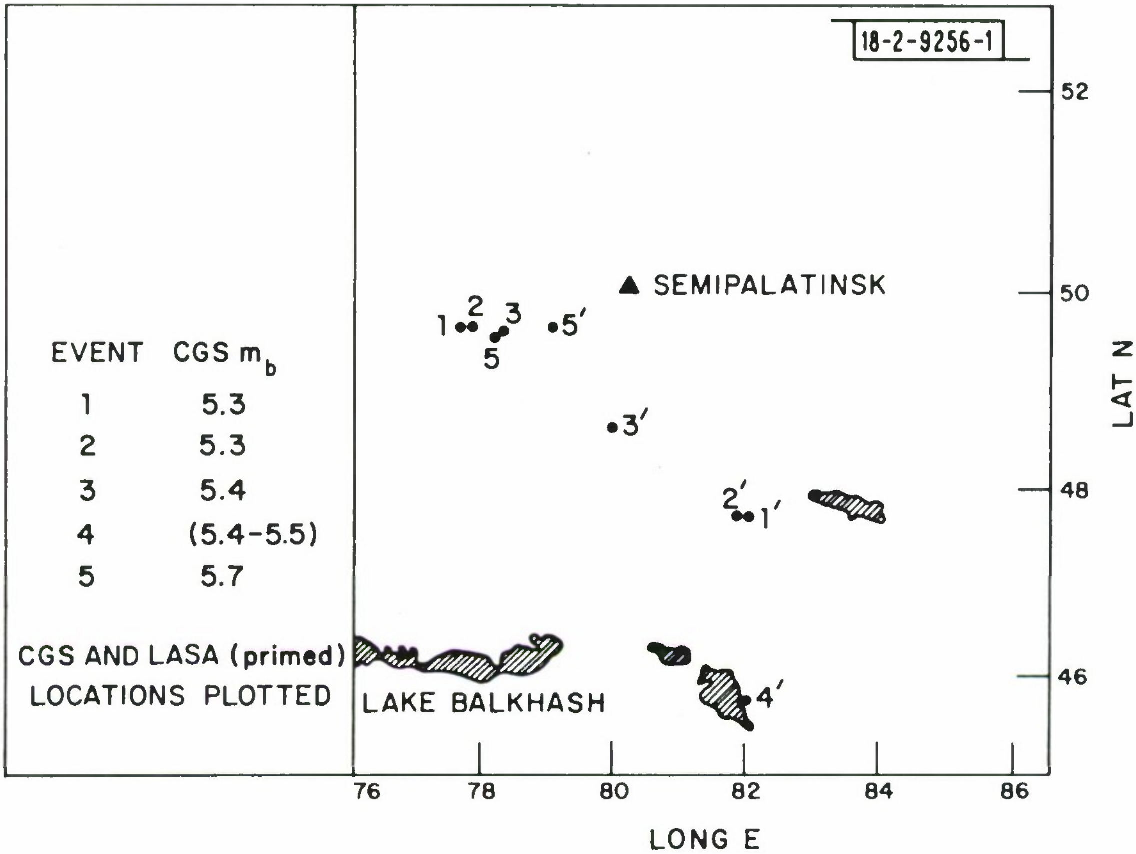

Fig. 5. Map of the source region showing the USCGS and LASA locations of the events used in this study.

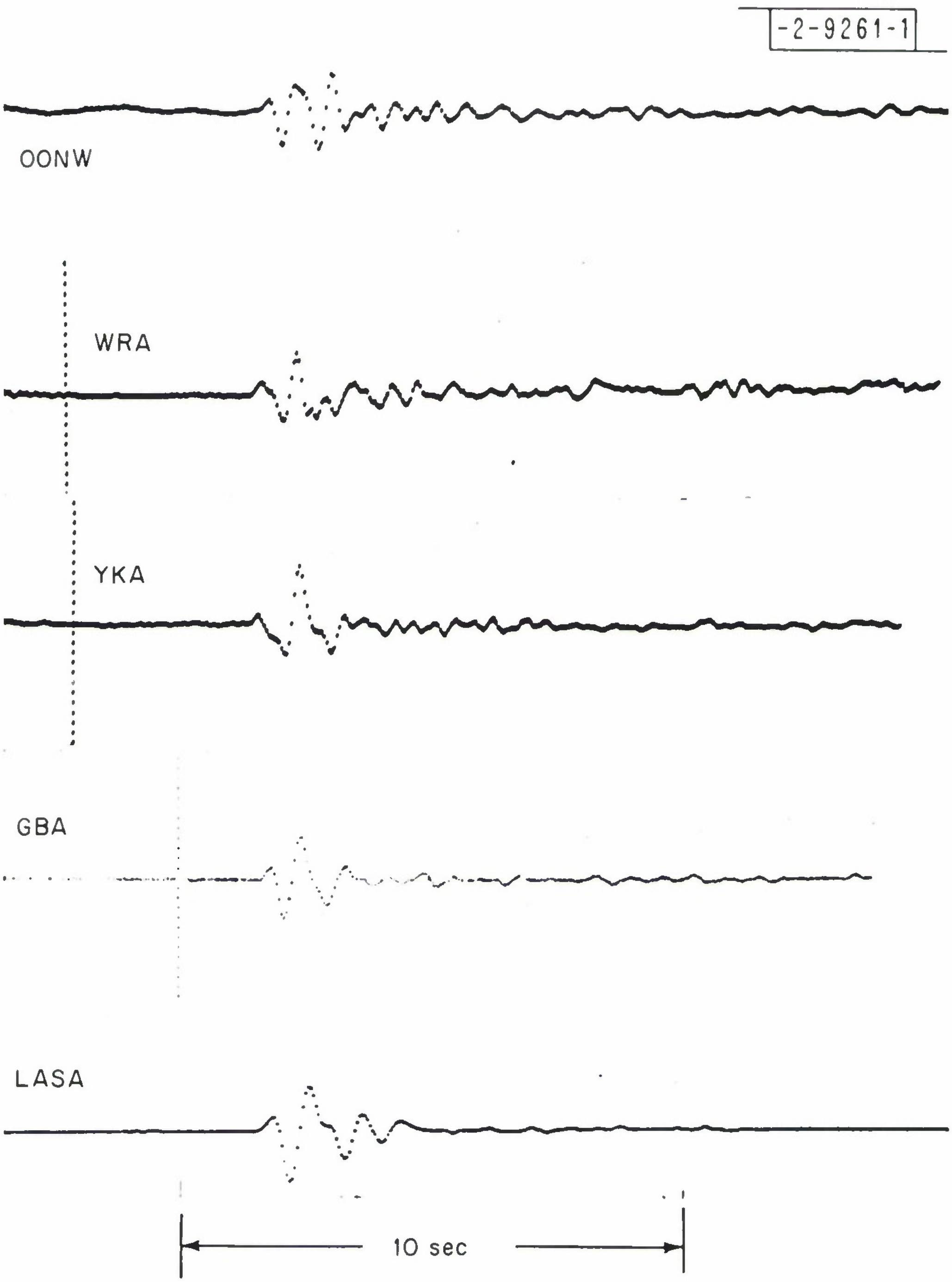

Fig. 6. The beams at the five arrays of event 1, $m_b = 5.3$.

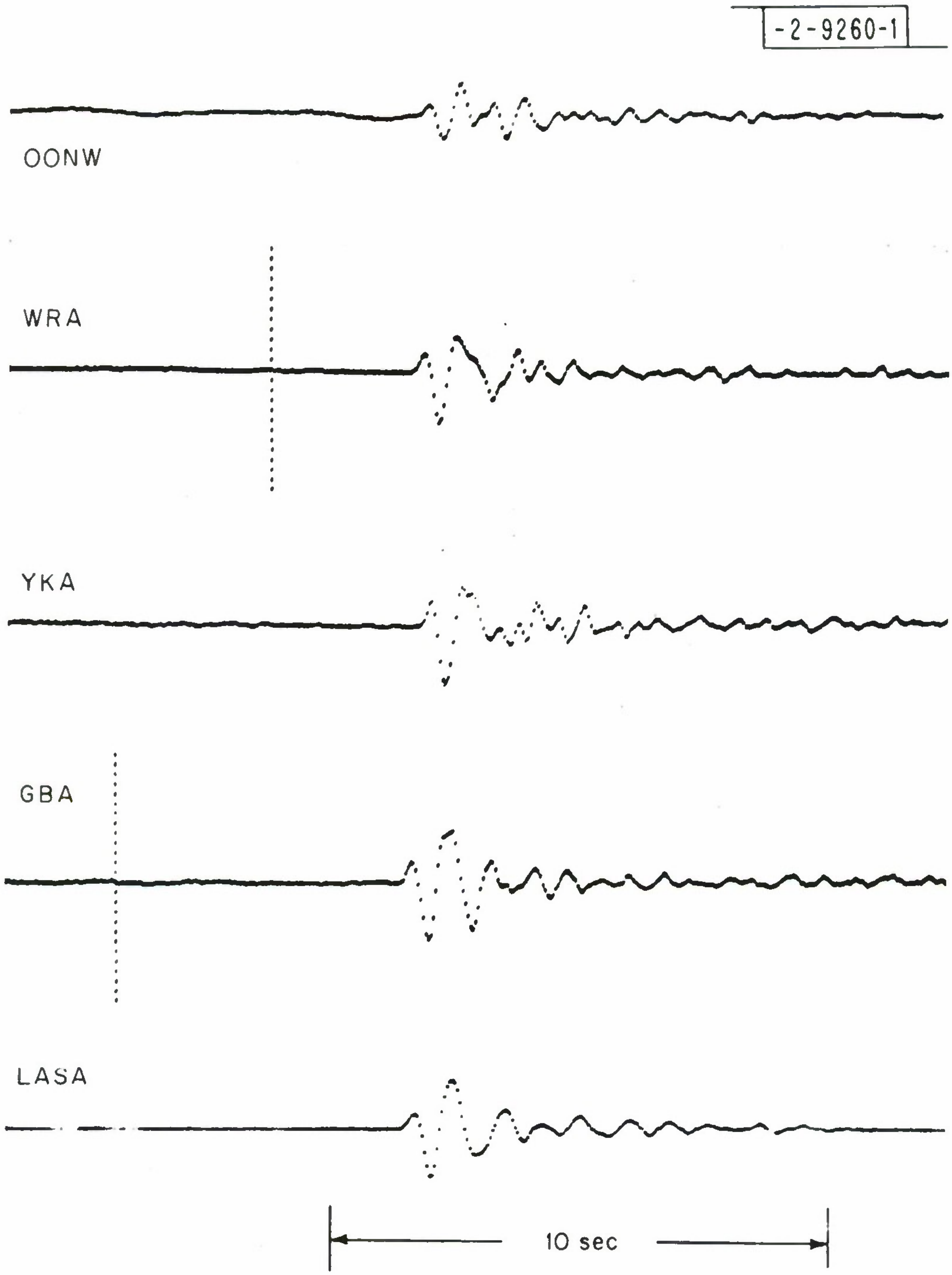

Fig. 7. The beams at the five arrays of event 5, $m_b = 5.7$.

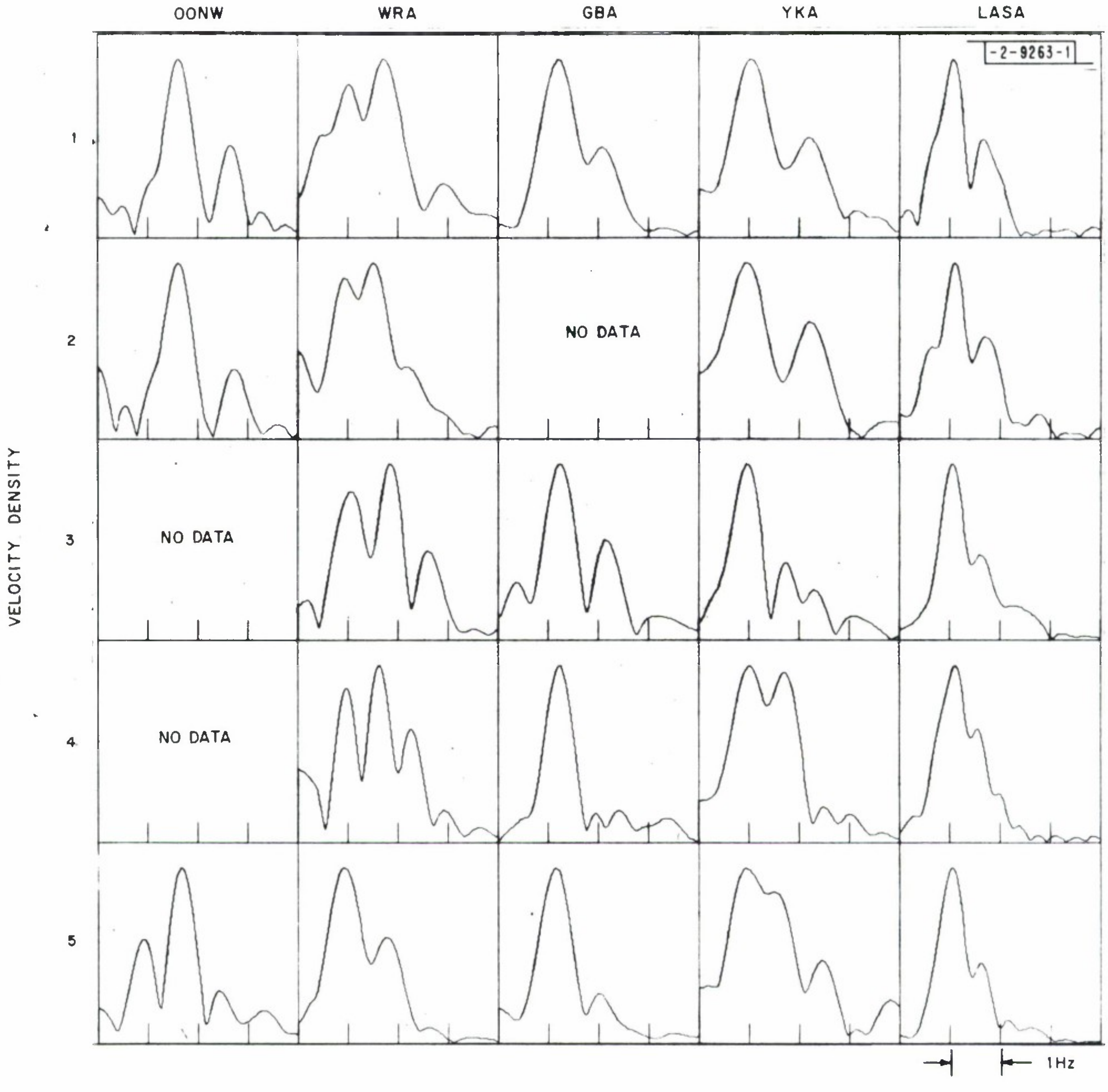

Fig. 8. The spectra computed using the first 3 seconds of beam data.

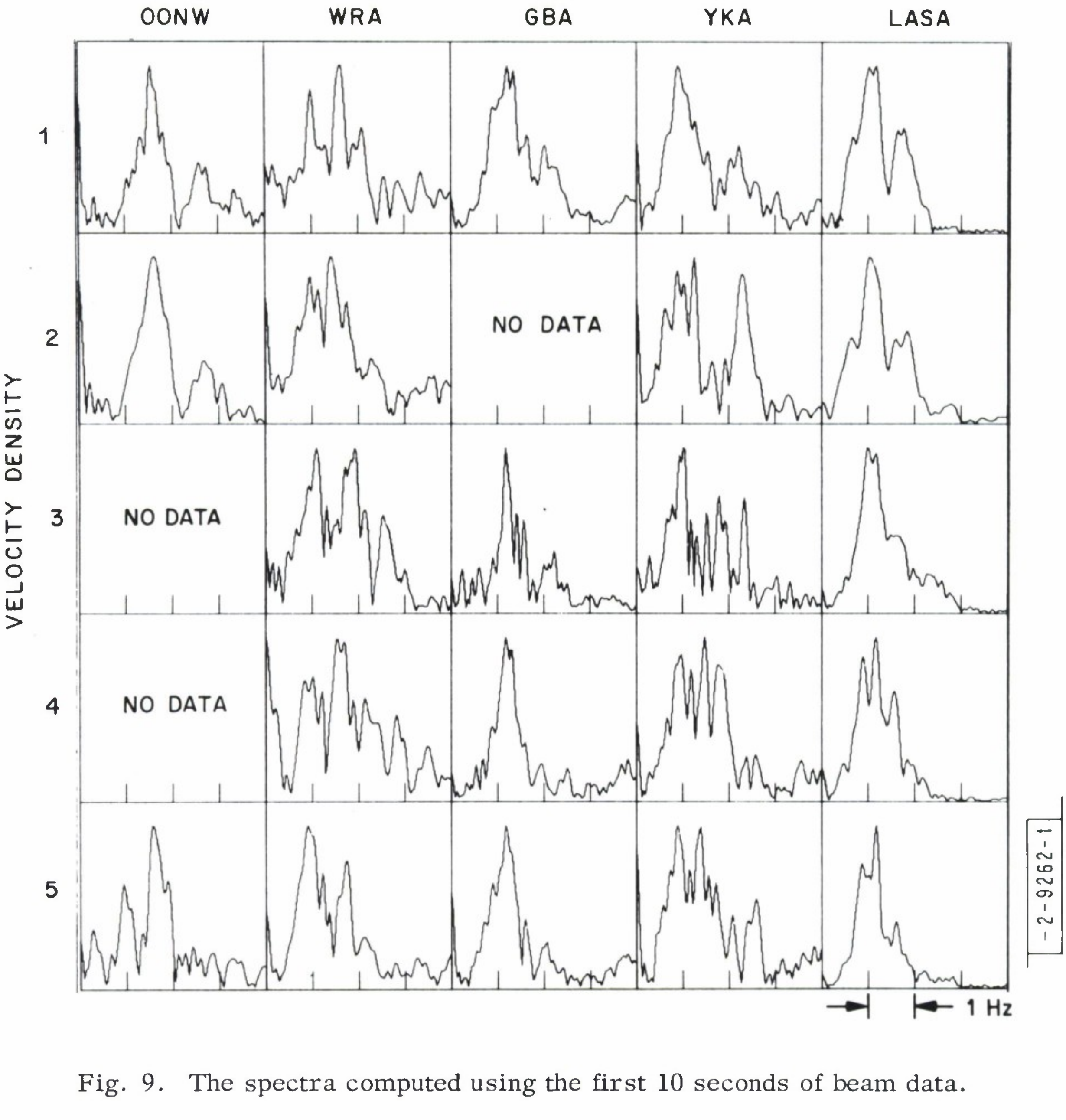

Fig. 9. The spectra computed using the first 10 seconds of beam data.

and 4.0 Hz, uncorrected for instrument response and individually normalized. The ordinate represents velocity density in the frequency bands considered here. In these figures, spectra of common events are numbered and plotted along a given row, and spectra common to a given station are plotted in a given column. Considering only major modulation and energy content there is little difference between the 10 second spectra and the 3 second spectra except the latter are somewhat smoother.

The most general observation to be made of the spectra in Figure 8 say, is that the spectra in any one column tend to look more alike than the spectra in any one row. That is, in general, recording site characteristics appear to dominate over source characteristics. Most of the spectra of Figure 8 show a minimum developed between 1 and 2 Hz. If this minimum is attributed to modulation caused by the delayed surface reflection near the source, where the frequency of the first minimum is at τ^{-1} Hz if τ is the total delay, delays of .5 to 1.0 seconds are inferred. This corresponds to a depth of burial-velocity of overburden ratios of about .25 to .5 seconds. The point is, these minima are not in the same place at each array for a given event and the variations are too large to be accounted for by variation of the take off angle with epicentral distance. Although the free surface above the source is the strongest velocity contrast encountered by the system of waves that arrives within the first 3 seconds; the effects of this reflection as predicted by plane wave, plane layer, elastic theory do not seem readily identifiable on this set of world wide data. Cepstral analysis preformed on these spectra confirmed this conclusion.

It is evident from Figure 9 that there is a shift in spectral content toward lower frequencies with an increase in magnitude. Although this shift is not always obvious

when any pair of adjacent spectra in any one column are compared, it is evident when the spectra of event 1 are compared with the spectra of event 5 at all array sites except YKA. What follows is an attempt to quantify these shifts if they exist and to combine the spectral data from all the array sites for a single event in some rational manner in order to gain a better estimate of certain source parameters.

III. ANALYSIS

Haskell's[2] algebraic expression for the far field spectral modulus of the radial component from an explosive source $|U(f)|$ is given in the equation (1).

$$|U(f)| = K \left\{ \frac{[1+(\alpha \beta f)^2]}{[1+(\beta f)^2]^5} \right\}^{\frac{1}{2}} \tag{1}$$

In equation (1) f is frequency in Hz, and the parameters K, α, β relate to $\Psi(\infty)$, B, and k given in Haskell's Table 1 as follows

$$\alpha = 1 + 24B,$$

$$\beta = {}^{2\pi}/k$$

$$K = \Psi(\infty).$$

Haskell gives values for B, k, and $\Psi(\infty)$ in four media (granite, salt, tuff, and alluvium) at 5 kilotons yield. As Haskell points out, α is a function of the medium alone and is determined by the overshoot of the pressure at the elastic cavity boundary with respect to the long term pressure that is maintained by the explosion. The parameter β is determined by the medium and the pressure rise time at the elastic cavity wall. The solution for the displacement due to a pressure pulse on a cavity wall in an elastic medium indicates that β should scale as yield $^{\frac{1}{3}}$. K is a function of the medium and the source volume and thus it is a direct function of yield.

It is noted that although equation (1) is simple in form and its extrapolation to greater yields is based on some rather elementary theory which does not include burial depth; it gives very good agreement to observations (Werth and Herbst[3]) made near to the source on instruments with response similar to those used in this study.

Figure 10 shows the displacement spectrum modulus $|U(f)|$ plotted as a function of frequency for various kilotonages using Haskell's parameters for granite. The strong variation as a function of yield is evident. In order to quantify the observed spectral variation, equation (1) has been fitted in a least squares sense to the observed spectra by varying the source parameters K, α, and β. This technique has been employed by Johnson and Bakun[4] using Nevada tests observed in California. In order to gain reasonable estimates of the source parameters through such a scheme, corrections for effects of propagation must be made. The largest correction is that for the attenuative properties of the earth which requires knowledge of the quality parameter Q as a function of geographic position and depth. Sato[5] in an extensive review of the available Q measurements lists values of P wave Q (Q_α) in the mantle ranging from 100 to 4000 over the period range .1 to 100 seconds. Oliver and Isacks[6] give evidence which indicates gross variation in the attenuation properties of a rather narrow but tectonically active slice of the mantle such as the Tonga-Kermadec arc. Given these uncertainties and variations, it is difficult to make teleseismic studies of the short period source spectrum unless attenuation along each path has been measured and unfortunately an absolute measurement of attenuation cannot be made without knowledge of the source spectrum.

The following technique is suggested as a solution of this quandary. In equation (2) the function G is minimized over m frequency components f_j at a given array. G represents the square of the differences between the observed spectral data $D(f_j)$, corrected for instrument response and exponential attenuation, and Haskell's source model.

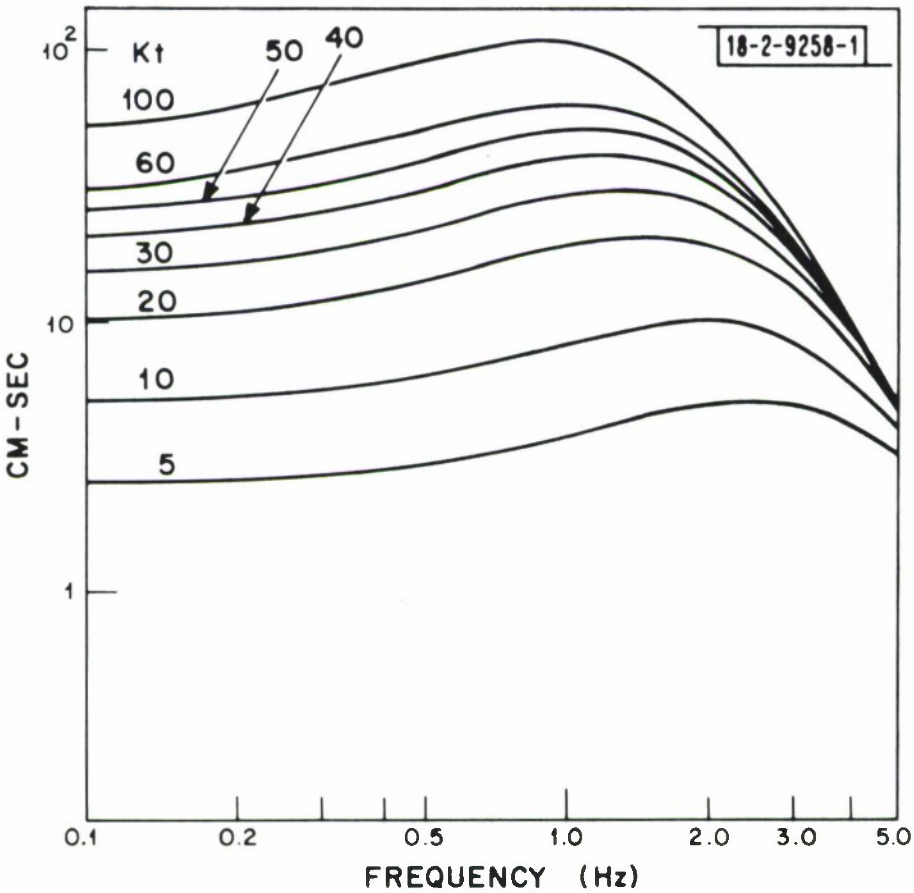

Fig. 10. Far field displacement density curves using Haskell's model for granite.

$$G = \sum_{j=1}^{m} [D(f_j)e^{+\pi t^* f_j} - KH(\alpha, \beta, f_j)]^2 \qquad (2)$$

The attenuation parameter t^* is defined as

$$t^* = \int_{ray} \frac{ds}{Q(s)\, v(s)} \qquad (3)$$

where ds is a ray element and v(s) and Q(s) are the P wave velocity and quality parameter at that element. G is minimized for each explosion at an array for various assumed values of t^*. Each value of t^* yields a suite of parameters K, α, β. Now Haskell's model implies that, for a given medium β will be proportional to $K^{\frac{1}{3}}$. Thus for each assigned t^*, log β is plotted against log K for all the events recorded at a given array and that t^* which shows the parameters varing most closely to the cubic law is accepted as the appropriate attenuation parameter from the source region to the array site. Table 1 lists the values of α and β determined at each site for each event for the accepted value of t^*. In this experiment the 10 second data was fitted between .6 and 3.0 Hz using a method described by Fletcher and Powell[7]. The values of the effective Q (Q_E, travel time divided by t^*) are also given to each array. Figure 11 shows the log plot of K vs β for the accepted value to t^* at each array. In each case the solid line has the slope of $\frac{1}{3}$ so that the variation of the fitted values of β vs K can be compared with the predicted variation. The scheme does not yield acceptable results with the YKA data however the parameters computed for t^*=.3 come closer to predicted variation than those computed for any other t^*.

It is difficult to assign quantative error ranges to the accepted value of t^*. The technique assumes source spectrum of a certain form, if this assumption is wrong

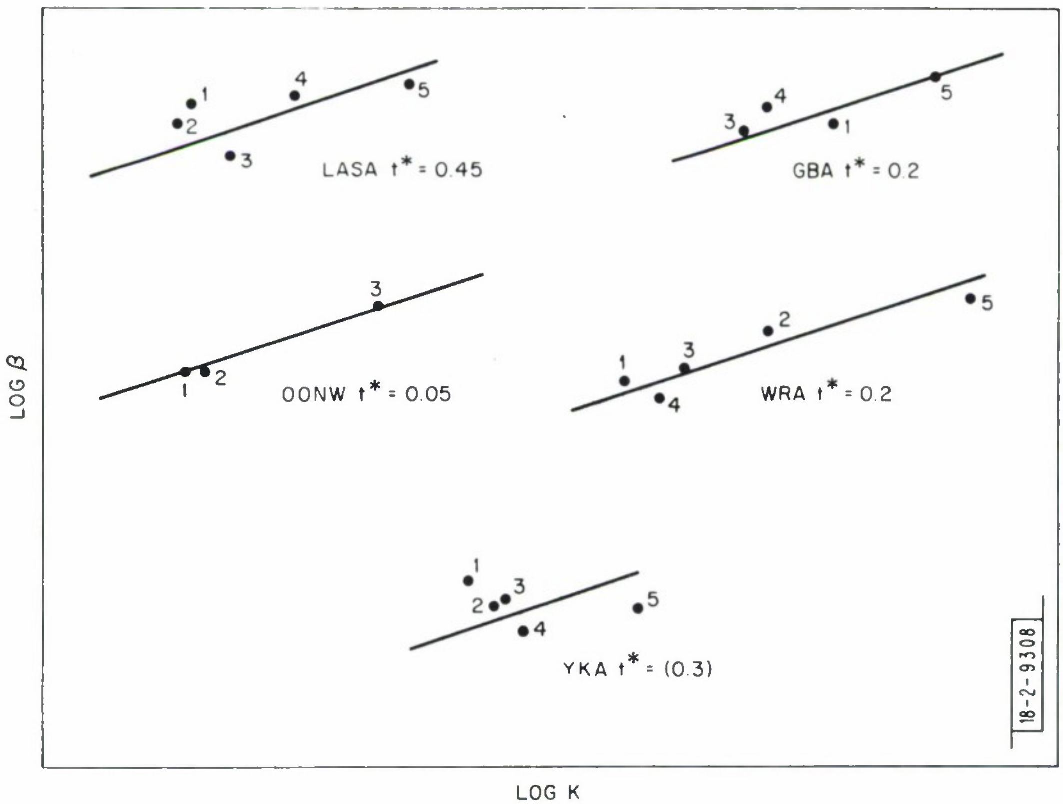

Fig. 11. A composite diagram showing the fitted values of β and K for the accepted values of t*. The straight lines have a slope of 1/3.

then the accepted values of t^* will be in error. It must be noted that, coincidentally or not, the values of t^* appear to be a direct function of array size. The attenuation of higher frequencies due to beamforming is difficult to access since it depends upon the coherence of these frequencies across an array and this coherence will be a function of array size. With this effect in mind, comparison of the t^* values computed for the U. K. and Norway sites is considered justified, however the value accepted for LASA is probably too large and should not be compared with those inferred at the smaller arrays.

Given these qualifications, some interesting geophysical conclusions can be drawn from the t^* determinations. OONW and GBA are approximately equidistant (38°) from the source region, yet the different paths to these sites show marked attenuation contrasts. Rays to each of these sites reach a maximum depth of about 900 km, however the path to Norway passes under the aseismic, stable platform of northeastern Europe, while that to GBA passes beneath the Hindu Kush of southwest Asia where earthquakes of depths up to 300 km are common. Since these sites are located on the Baltic (OONW) and Indian (GBA) shields, it is reasonable to associate the greater attenuation observed at GBA to the upper mantle within a seismically active region rather than to the crust immediately beneath southern India.

The values of α and β listed in Table 1 are all reasonable however they show a certain amount of scatter at each array for a given event and the values of β do not increase smoothly with magnitude considering all the events at a given array. In an attempt to obtain a better estimate of the source parameters α and β, the function G' of equation (4) was minimized.

TABLE 1

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SITE $Q_E \approx$		OONW (8000)	WRA (3600)	GBA (2000)	YKA (2200)	LASA (1600)	Σ
1	α	6.8	6.8	6.7	8.1	6.8	6.7
	β	0.42	0.53	0.59	0.71	0.57	0.53
2	α	6.8	6.6	NO	6.7	6.7	6.2
	β	0.42	0.67	DATA	0.64	0.52	0.51
3	α	NO	8.6	6.7	6.7	6.8	7.0
	β	DATA	0.56	0.58	0.66	0.44	0.54
4	α	NO	6.9	6.7	6.8	6.7	6.9
	β	DATA	0.48	0.63	0.55	0.59	0.57
5	α	6.7	6.5	6.5	6.7	6.7	6.4
	β	0.59	0.86	0.73	0.62	0.63	0.67

Table of fitted source parameters α and β for the accepted value of t* for events 1 - 5. Q_E is computed from travel time/t*. The values of α and β in the column under Σ were gained by fitting all the observed spectra for a given event simultaneously.

$$G' = \sum_{i=1}^{n} \sum_{j=1}^{m} [D_i(f_j)e^{+\pi f_j t_i^*} - K_i H(\alpha, \beta, f_j)]^2. \tag{4}$$

G' represents the square of the difference of the corrected observed spectral components and Haskell's model over m components at n arrays. The data $D_i(f_j)$ were corrected for instrument response and attenuation using the values of t* previously determined. Such a procedure assumes a spherically symmetric radiation pattern at the source, a justifiable assumption if these events are explosions. The results of this multi-site fitting scheme are shown in the right hand column of Table 1, under the symbol Σ. In this case the fitted values of β increase somewhat more smoothly with magnitude.

Experiments conducted with the fitting scheme indicate that the parameter α is less well determined than β; that is, the fitted value of α depended somewhat on the initial value assigned while β did not. Regardless of the starting value, the fit of the individual spectra would always converge for $4 \leq \alpha \leq 9$, at acceptable values of t*. Haskell's values for this parameter range from 12.8 for alluvium to 2.2 (arbitrarily assigned) for tuff, with intermediate values of 6.9 and 5.0 for granite and salt respectively.

Although more well determined than α, it is difficult to assess the significance of the β values gained in the simultaneous fit, since the yields of these explosions are unknown. Most recent magnitude-yield relations (Evernden[8]) are based on P_n measurements and directly applicable only to specific source and receiver regions of certain geologic types. Since the source and receiver regions used in this study have not been calibrated to the regions used by Evernden, an absolute estimation of yield from magnitude, using his data, is not allowed. Davies[9]

(Figure 3.1) plots m_w versus yield for a series of explosions varying from about 4 to 100 kilotons. His m_w is a body wave magnitude determined on a short period WWSSN type instrument and his data can be fairly well represented as a straight line by

$$m_w = 3.8 + \log Y. \tag{5}$$

Here it is felt that m_w is more akin to the USCGS m_b and equation (5) is used to test the results obtained in this study. A rough test that can be applied is to compare the USCGS m_b values, which have yet to be used in this analysis, with the values of β determined by minimizing (4). Roughly speaking, this is a "world wide average magnitude" compared to a "world wide average β". Using the values of β determined by (4) (listed under Σ in Table 1), $\log_{10}\beta^3$ is plotted against USCGS m_b in Figure 12. Recalling that β^3 should be proportional to yield, these points are compared with a line which has a slope of 1.0 in Figure 12. Haskell's model and parameters for granite would predict a slope of 1.0 for such a plot in this magnitude range.

Finally, there exists one other population of data against which to test these suggestions of source information in the teleseismic, short period explosion spectrum. Lacoss[10] has computed a spectral ratio (SR) for some 23 Semipalatinsk events observed at LASA. Using the array beam, he defined SR as

$$SR = \sum_{f_j=1.45}^{f_j=1.95} v(f_j) \Big/ \sum_{f_j=.35}^{f_j=.85} v(f_j) \quad , \tag{6}$$

a ratio of the sum of components in a high frequency band (1.45 - 1.95 Hz) to those in a low frequency band (.35 - .85 Hz). Lacoss recognized a trend in the SR measure-

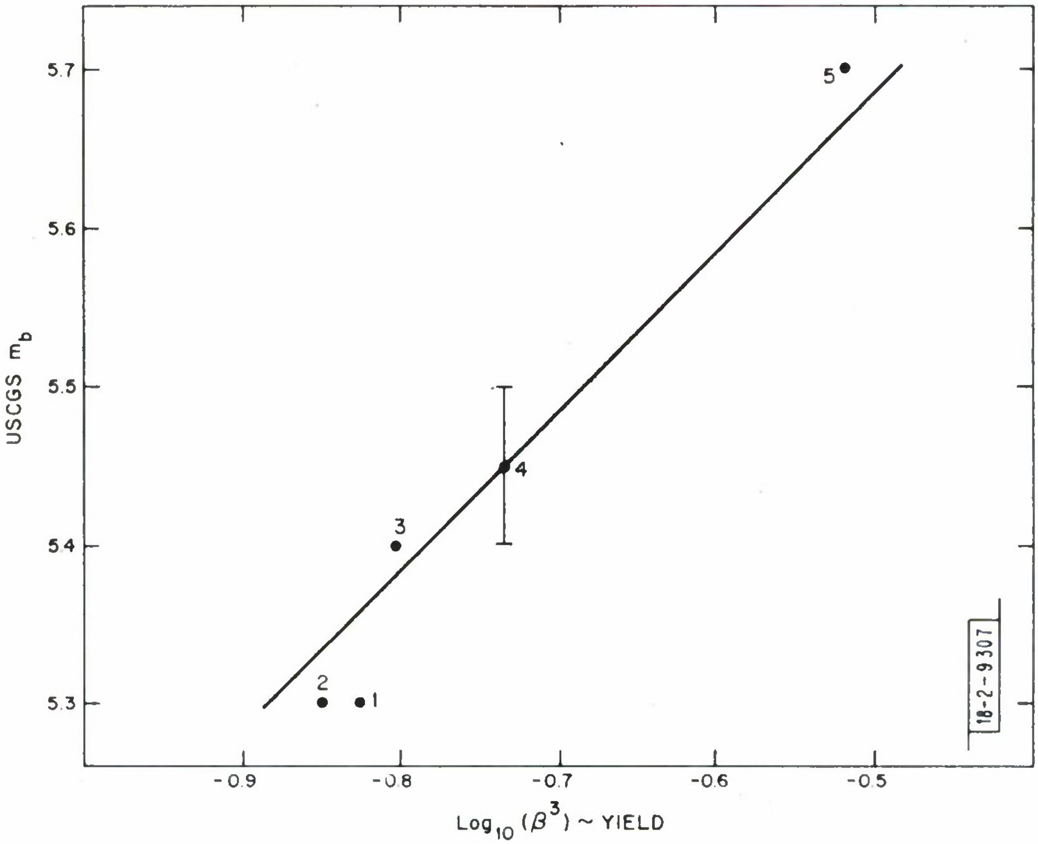

Fig. 12. USCGS m_b plotted vs $\log_{10}(\beta^3)$ using β determined in the simultaneous fit of all the spectra of each event. The straight line has a slope of 1.0.

ments, the values of SR measured apparently increased with a decrease in body wave magnitude (m_b). In order to test the significance of this trend, Haskell's values for granite, $t^* = .45$, and the magnitude-yield relation of equation (5) have been used to compute SR as a function of magnitude. The results of this test are given in Figure 13, where the computed SR is plotted as a solid line and those measured are plotted as dots. Here it is seen that the assumed source model and the measured attenuation parameter appear to offer an explanation for the observed trend in the SR measurements at LASA.

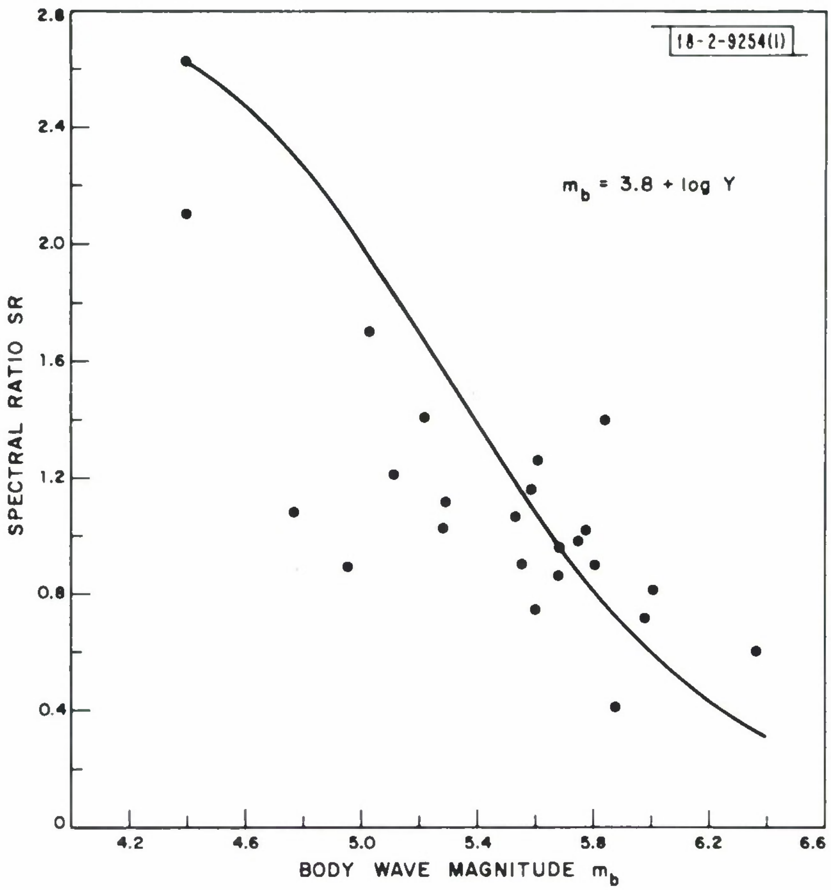

Fig. 13. Lacoss' spectral ratio data at LASA plotted vs m_b. Solid line is computed using Haskell's model for granite and $t^* = .45$.

IV. CONCLUSION

The primary conclusion of this study is that there exists information concerning the nature of the explosive source in the short period spectrum as observed at teleseismic distances. I cannot say that Haskell's model should be preferred over another or that the attenuation parameters assigned to the beam spectra represent an accurate absolute determination of t^* for a specific path. It is asserted however, that obvious variations in short period spectrum as a function of source size have been quantified and world wide data combined in a rational manner to give an estimate of the parameter which controls this variation.

ACKNOWLEDGEMENTS

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13. ABSTRACT

A study has been made of the short period spectra of five presumed explosions recorded at five arrays. An attempt has been made to relate contrasts in spectra of different events recorded at the same site, to source size; and contrasts observed at different arrays for a given event, to the earth's attenuative properties. Haskell's model for the explosion spectrum was fitted to each event individually after corrections for instrument response and various exponential attenuations. At a single array, that attenuation which allowed the fitted parameters to vary as dictated by the model was chosen as the correct one. With the attenuation estimated to each array, the spectra observed at all the arrays for a single event are fitted to a source model simultaneously. In most cases the individual and simultaneous fitting schemes yield reasonable values for the source parameters. Haskell's model and the estimated attenuation parameter for a central Asia to LASA path apparently explains a trend in short period spectral ratio measurements as a function of magnitude.

14. KEY WORDS

seismic array	LASA
attenuation	spectral analysis